## RMTM $\Phi$-Fund ${ }^{\text {TM }}$ Explanation

## 1 Background

Historically, accounting systems have all been limited by a single model for establishing the value of an asset: price times quantity $\left(\mathrm{P}^{*} \mathrm{Q}\right)$. This mechanism is broadly effective, but also highly limiting. Without realizing it, the "P*Q" model restricts thinking to developing products and services that conform to this two-variable model. Because no accounting systems exist to support them, no products or services have been devised that require more variables. It is like the Japanese Kanji alphabet. If the Japanese language were limited to the Kanji characters, the language could not adapt to innovations that occurred once the alphabet was established. To overcome this, the Japanese use an additional alphabet, kata kana, which allows them to express concepts like the Java programming language or English names.

An example of an application that requires a new accounting system is the multiplier mutual fund. Accompanying the growth of index funds has been an expansion in aggressive, leveraged funds whose value is linked to an underlying index, but whose daily change is calculated as a multiple of the daily percentage change of the underlying index (henceforth referred to as multiplier funds). While on an intra-day and daily basis such funds closely mirror the performance of the underlying index, the nature of their $\mathrm{P} * \mathrm{Q}$ pricing mechanism is such that over time, the funds lose fidelity with the underlying index changes. This loss of fidelity is caused by the natural changes in direction (e.g., from rising to falling or vice versa) in the value of the underlying index. This creates a divergence between the fund and its underlying index. This divergence means that an investor may incur a loss in the value of a fund even though the underlying index is equal to or better than the value point at which the fund was purchased. Over time, as daily index fluctuations manifest multiple changes in direction, this performance gap can continue to grow. as is illustrated in Figure 1.

Figure 1 shows the values of an imaginary index over a period of 20 trading days. Also displayed is the value of an imaginary multiplier mutual fund whose daily percentage change is double the daily percentage change of the underlying index. If an investor purchased the ETF on Day 1, when the index was at 100 , he would pay $100^{1}$ for the index fund. Ten days later, when the index had risen and then dropped back to the original purchase price of 100 , the multiplier fund, rather than also returning to its original purchase price of 100 , is valued at 99.19 , nearly $1 \%$ less than its original purchase price. On day 20, after the index continued to decline and then rose back to the original purchase price, the multiplier fund is even further below the initial purchase price at 98.44 , reflecting a paper loss of $1.5 \%$ even though there has been no change in the underlying index. Even though the index is virtually unchanged, an investor who sells at this point has lost money. This divergence would continue to increase as long as the underlying index does not make a sustained and dramatic move in any one direction. Only when the underlying index begins a steady and dramatic move upwards does the investor overcome the negative divergence and exploit the benefits of the aggressive ( 2 x ) multiplier fund.

[^0]Figure 1: The diverge of multiplier funds from their underlying average tends to increase over time.


Figure 2 displays the value of an imaginary multiplier mutual fund whose daily percentage change is inversely double the daily percentage change of the underlying index. If an investor purchased the inverse multiplier fund on Day 1, when the index was at 100, he would pay 100 for the index fund. Ten days later, when the index had risen and then dropped back to the original purchase price of 100 , the inverse multiplier fund, rather than also returning to its original purchase price of 100 , is valued at 97.65 , nearly $2.5 \%$ less than its original purchase price. On day 20, after the index continued to decline and then rose back to the original purchase price, the inverse multiplier fund is even further below the initial purchase price at 95.45 , reflecting a paper loss of $4.5 \%$ even though there has been no change in the underlying index. Again, this divergence would continue to increase as long as the underlying index does not make a sustained and dramatic move in any one direction. Only when the underlying index begins a steady and dramatic move downwards does the investor overcome the negative divergence and exploit the benefits of the aggressive (2x) inverse multiplier fund.

Figure 2: Inverse multiplier funds diverge even more than standard multiplier funds.


While the illustrations in Figures 1 and 2 focus only on the value of the fund when the underlying index returns to the value at which the fund was purchased, over time the divergence will frequently increase to the point that even when the index has moved in the desired direction (up for a positive multiplier fund and down for an inverse multiplier fund), the value of the fund can be below its original purchase price as is illustrated in Figure 3.

Figure 3: The performance gap can grow so large that an investor is "under water" even when the underlying index has moved in the predicted direction.


Figure 3 illustrates the comparative values of the Dow Jones Industrial Average (DJIA) with a two hypothetical multiplier mutual funds over the period 09/10/98 through 2/2/09. The gray line represents the value of the underlying DJIA index relative to its value on 09/10/98. The green line represents a $2 x$ multiplier fund. The red line represents a $-2 x$ multiplier fund. (For reference purposes the black line displays the actual value of the DJIA using the scale on the right. Note that the DJIA begins the period at approximately 8000 and ends the period at approximately the same value.) Note that while the relative value of the DJIA (gray line) spends little time below $100 \%$, the $2 x$ fund spends considerable time at or below the gray line. And even though the DJIA ends at the same value it began, investors in the $-2 x$ fund would have lost three-fourths of their investment. Even when the DJIA is negative (around 09/10/02), the $-2 x$ fund displays a loss of nearly $30 \%$.

Also displayed in Figure 3, the blue line represents the sum of the value of the $2 x$ and $-2 x$ funds. Ideally, this value should be zero because any gains in the $2 x$ fund should be equal to losses in the $-2 x$ fund. But the blue line spend considerable time below $100 \%$ and ends the period at only $50 \%$-- reflecting the cumulative divergence of both funds..

Many investors in aggressive multiplier funds are day traders who remain unaffected by the divergence that only arises over time between an underlying index and funds that track the index using a multiplier (other than 1x). But for those investors who are seeking to exploit market trends beyond 1-day, current multiplier funds can impose a significant penalty for positions taken over a period of time. The shortcoming that leads to this performance gap between the multiplier fund and its underlying index arises because the change in value of an individual's holdings is tracked using a single variable: share price. This problem cannot be fixed by merely changing some underlying algorithm. Indeed, the algorithm for pricing the value of a share of the multiplier fund is fixed by the stated investment objective of the fund.

The performance gap problem arises as a result of the traditional $\mathrm{P}^{*} \mathrm{Q}$ model (Price times Quantity) used for valuing mutual funds. In the traditional $\mathrm{P}^{*} \mathrm{Q}$ model, Price is the only
"variable." Quantity is considered to be constant. If you purchase 100 shares of multiplier fund X, your holdings remain at 100 shares until such time as you either purchase more or sell some of your shares. The change in value of your holdings is reflected solely in the daily change in the Price of each share. Using this single variable for establishing the value of one's holdings does not provide enough degrees of freedom to both reflect the daily multiple of change in the underlying index and maintain fidelity with this index.

Thus, a need exists for an entirely new method for valuing a multiplier fund to exploit its inherent aggressiveness without incurring the performance gap.

## 2. $\Phi$-Fund ${ }^{\text {TM }}$ Solution

The value of holdings in a mutual fund is apportioned over more than one variable to provide for both accurate valuation of a multiplier fund that is consistent with its advertised aggressiveness (multiplier) and fidelity with the underlying index. Currently, the entire value change is reflected in the price.

In one embodiment, a mechanism includes spreading the daily change in value of a multiplier fund over at least two variables. By apportioning the change in value between the price of each share of the fund and the number of shares held such that the share Price continuously maintains a 1:1 relationship to the value of the underlying index, the value of an individual's holdings in the fund retains its fidelity with the underlying index.

Figure 4 demonstrates the better fidelity of this embodiment compared to a traditional multiplier fund. Figure 4 displays the same 20-trading-day period of the underlying index in black. It shows a typical 2x multiplier ETF in light blue with its divergence. As well this chart shows a $2 x$ multiplier fund using the subject invention in dark blue. There is no divergence when the underlying index returns to its initial price of 100 .

Figure 4: The subject embodiment maintains its fidelity to the index over time.


## Detailed Description

Using the subject accounting system, a mutual fund valuation engine can be configured for a multiplier fund to accurately reflect a single trading day's percentage change while also retaining long-term fidelity with gains or losses in an underlying index. Specifically, the valuation engine
apportions the change in value over more than one variable. This contrasts with current valuation engines which express all changes in terms of a single variable: price.

In one embodiment, the number of shares can be held constant, but the share price is divided among two variables. The first variable is the value of the underlying index. The additional variable, $\Phi$, is introduced. The value of $\Phi$ is a calculated value described by the equation

$$
\Phi=\left(\left((1-\mathrm{M}) * \mathrm{IV}_{\text {Index }}\right)+\left(\mathrm{CV}_{\text {Index }} * \mathrm{M}\right)\right) / \mathrm{CV}_{\text {Index }}
$$

where
M represents the multiplier for the mutual fund (e.g., $-2,-1,2,3$ ), and
$\mathrm{IV}_{\text {Index }}$ represents the initial value of the underlying index (e.g., 100)
$\mathrm{CV}_{\text {Index }}$ represents the current value of the underlying index.
For a 2 x multiplier fund with an initial index value of 100 , if the value of the index rises to 103 , the value of $\Phi$ would be $(((1-2) * 100)+(103 * 2)) / 103=((-1 * 100)+206) / 103=102.91$.
The share price of the example 2 x mutual fund is then the product of the index times $\Phi$. In the above example, this would be $103 * 102.91=106$, manifesting exactly a 2 x change versus the underlying index.

The advantage of splitting the share price of the multiplier mutual fund into two components (e.g., the index value and $\Phi$ ) is that both values can be tracked for all transactions and reported along with the resulting share price independent of the number of shares. For this reason, $\Phi$ is investor independent - allowing the multiplier fund to be traded dynamically (e.g., as an ETF), as long as $\Phi$ is reported along with the index value at the time of the trade to determine the ultimate settlement value of the trade. Because the baseline for $\Phi$ is established based on the initial index value, the calculation of $\Phi$ and the resulting settlement value of the trade can be easily automated because it moves in tandem with the index value. $\Phi$ is a function of the underlying index. For any particular index value during a particular trading day, there is one and only one value for $\Phi$. Unlike current leveraged funds, no matter what path the index traverses over time, when the index is 103 , $\Phi$ will always be 102.91 , and the value of the share price of the investment will always be 106; there is no "decay" from changes in the direction of the path of the index.

Figure 5 displays example values of $\Phi$ for various multipliers $(+3,+2,-1,-2,-3)$ of a fund whose underlying index ranges from 90-160. The functions continue over expanded ranges.

Figure 5: There is one and only one value of Phi for any value of the underlying index, regardless of what multiplier is used.


## 3. Implementing the RMTM Ф-Fund ${ }^{\mathrm{TM}}$ : A Simple Example

The key to the $\Phi$-Fund ${ }^{\mathrm{TM}}$ solution is utilizing a valuation function for $\Phi$ that is a function of the underlying index. One such function that is particularly easy to understand and implement would be a fund whose value varies as a multiple of the nominal change in the underlying index, rather than the percentage change. Such a fund would never require re-indexing. If the index rises from 100 to $105,2 \mathrm{X}$ fund would rise from 100 to 110 (the same as with a percentage-based fund). But when the index then rises from 105 to 110 , the nominal-change fund would increase from 110 to 120 (while the percentage change fund would increase a bit more). In the index then drops back to 100 , the fund drops back to exactly 100.

There are two issues with a nominal-change based fund:

1. The leverage provided by the fund varies continuously with its purchase price
2. The value of the fund could go below zero

The first problem is minor, but requires some clarification. The second problem requires a solution, but the $\Phi$-Fund ${ }^{\mathrm{TM}}$ approach can accommodate it. Each issue is described separately below.

## Variable Initial Leverage

Current leveraged index products that leverage the percentage change in the underlying index (e.g., ProShares 2X ETFs and Direxions 3X ETFs) offer exactly 2X and 3X leverage on the date of purchase. However, this leverage holds for only one day. After one day, because of reindexing the leverage received upon closing out a position is highly volatile. In fact, the leverage can actually be negative if the fund is held over a period of volatility.

Creating a fund that applies leverage to the nominal change in the underlying index implies that the leverage offered at the time of initial purchase will vary. If a 2 X fund opens at 100 , it will provide exactly 2 X leverage when (using the example above) the index moves to 105 (up 5\%) and the fund moves to $110(10 \%)$. But in a subsequent move of the index from 105 to 110 ( $4.8 \%$ ) while the fund moves from 110 to $120(9.1 \%)$, the 2 X nominal change represents only 1.9 X leverage. The good news, however, is that this leverage will remain constant until the fund is sold. Regardless of the path of the index, the investor who purchases the fund at 110 will always received 1.9 X leverage when he sells. Because investors are typically more concerned about the actual leverage they receive when they close out their position, this is actually an advantage of the nominal-change fund.

## Keeping the Value of the Fund Positive

Keeping the value of the fund from going negative is a problem for nominal-change funds. If, for example, the index that underlies a -2 X inverse fund rises more than $50 \%$, the inverse fund will be negative.

Current funds address this challenge in one of two ways. The first way is to force a stop-loss transaction. ETNs issued by Invesco Powershares terminate if their value ever touches zero. Barclays has ETNs that use a similar approach, terminating when their value declines $80 \%$ to allow more room for major market dislocations.

The second way is to re-index the fund if-and-only-if, its value declines below a fixed demark. Credit Suisse has marketed ETNs that re-index if their value declines $20 \%$ from their opening value. They will continue to re-index for every subsequent drop in value. But by re-indexing, they become subject to path dependence and value erosion.
$\Phi$-Funds ${ }^{\mathrm{TM}}$ do not require either a stop-loss mechanism or re-indexing. They can be kept from going to zero merely be selecting a function for $\Phi$ that will never go to zero. In fact, a composite $\Phi$ function can be used that uses a nominal change valuation but then switches to a function that has zero as an asymptote at a predefined value. In this way, the value of the fund remains fully known and predictable in advance for any value of the underlying index.

Figure 6 illustrates the behavior of the various fund structures discussed when the value of their index steadily declines. The gray line represents the underlying index. The blue line represents a typical 2X fund that reindexes based on time (e.g., daily), such as a Proshares fund. The pink line represents a 2 X nominal change fund that could go negative and points out the place at which Barclays implements their stop-loss termination. The orange line illustrates products that re-index based on value (rather than time), such as the Credit Suisse ETN. The green line represents a $2 \mathrm{X} \Phi$-Fund ${ }^{\mathrm{TM}}$ that incorporates $\Phi$ function that causes it to never go to zero. Unlike the standard 2X fund that it appears to mirror (blue line), the $\Phi$-Fund ${ }^{\mathrm{TM}}$ will retrace its steps exactly when the index begins to rise. It always maintains fidelity with its underlying index.

Figure 6: A comparison of the different fund constructs in a steadily declining market for their underlying index.


## 4. Conclusions

Using the $\Phi$-Fund ${ }^{\mathrm{TM}}$ approach provides the following benefits:

## For fund managers:

1. An opportunity to create a leveraged index fund that targets a currently unserved market segment: buy-and-hold investors.
2. The ability to create a fund with "sticky" assets resulting from the longer term purchases by buy-and-hold investors.
3. The ability to create a fund that cannot be copied because of patent protection of the $\Phi$ Fund ${ }^{\mathrm{TM}}$ technology.
4. The ability to gain some protection from the aggressive fee cutting among fund managers because of the limited exclusivity afforded by $\Phi$-Fund ${ }^{\mathrm{TM}}$ patent protection.

## For investors:

1. The opportunity to invest in low volatility indices while still obtaining higher leveraged returns.
2. The opportunity to invest in leveraged index funds without concern for the timeframe over which the investment is held.
3. The opportunity to invest in products with leveraged returns without the risk of margin calls.
4. The ability to invest in products with leveraged returns, suitable for holding in a retirement account.
5. The ability to bet against the market by investing in products with inverse leverage, suitable for holding in a retirement account.

[^0]:    ${ }^{1}$ For convenience of illustration, the initial values of both the index and the ETF are set at 100 . There is no need for these values to be the same. But this allows for easy comparison of percentage change.

