A New Approach To Leveraged Index Funds

Abstract

The ΦFunds™ leveraged-index fund structure is presented that does not require re-indexing and does not suffer from path dependence. As such, it is appropriate for long-term investment, including being suitable for retirement accounts, because investors will never lose more than their initial investment. By applying this leveraged fund structure to an index, such as the S&P500 or Dow Jones Industrial Average, that are designed to rise over the long-term, investors can obtain outsized returns that consistently beat market averages over the long term.

1. Introduction

This article addresses a novel method of structuring leveraged index funds (both mutual funds and Exchange Traded Funds (ETFs)) to remove their path dependence, making them suitable for long-term investment, including retirement accounts. The article describes the path dependency problem with current leveraged index funds and presents a solution that enables even conservative investors to avail themselves of leveraged returns while making long-term investments in such indices as the S&P500, the Dow Jones Industrial Average, or the NASDAQ. Because these indices are designed to continuously rise over the long term (because laggard companies are frequently removed and replaced by growing companies), long-term investors can be expected to obtain outsized returns without incurring outsized risk.

2. Background

Leveraged Index ETFs have been around for more than a decade. They enable speculators to invest in standard indices such as the S&P 500 and the Dow Jones Industrial Average (among others) and obtain returns that are multiples of the returns accruing to their underlying index. Speculators can even bet against these indices by purchasing inverse funds that provide positive returns when the underlying index declines.

The way these funds work is based on the percentage change in the underlying index. If the index rises 2% on a particular day, a 2X leveraged index fund would rise 4%. A further benefit of this approach is that they can never go below zero. Since the underlying indices will always have a positive value, the most an index could decline is 99.99%. And realistically, such a dramatic loss in products based on broad market indices such as the S&P500, Dow Jones Industrials, NASDAQ, etc. rarely see declines of more than 20%. This makes investing in such funds preferable to obtaining leverage by buying on margin, because there are no interest payments due and no margin calls.

The promise of these funds is the ability to obtain leveraged returns on investments in traditional indices that historically rise over the long term without the risk of a margin call. this prompted many investors to jump into them when they were first offered. But early investors – many of whom failed to read and understand the prospectus for the ETF they

purchased – were shocked by the disappointing results that they achieved in the volatile markets during the 2008 decline.

3. The Problem

Exciting as are the prospects of earning outsized returns when correctly judging whether the market will move up or down, all current leveraged index funds suffer from a shortcoming that is inherent in the way they are structured. They are all path dependent. What this means is that the value of the ETF loses fidelity with the underlying index after a short, predetermined period – typically one day for most US ETFs. This is the result of simple arithmetic.

For example, imagine a notional index is at 100 and a 2X ETF is also at 100 on the morning it is purchased. At the day's closing bell, the index rises 10% to 110. The ETF would then rise 20% to 120 – a tidy gain. The next day, the index returns to 100 (a loss of ~9%). The ETF then declines just over 18%. But 18% of 120 is $21.82 -- more the $20 gain the previous day. The ETF is now below its initial purchase price (98.18) – even though the underlying index is exactly where it was when the investment began. There is no smoke and mirrors involved. It is just arithmetic. Over longer periods of time, particularly over periods of extreme volatility in the underlying index, this problem can grow.

This loss of fidelity over time means that if you purchase a 2x leveraged index ETF when the underlying index is at 100 and over several weeks the index rises to 110, the return on the investment will depend on the path the index took to get to 110. If it went up steadily -- a bit every day until it reached 110 -- the return will actually exceed 2X. However, if the path from 100 to 110 was a volatile one with dramatic ups and downs, the return will be less than 2X. In fact, in volatile periods (e.g., major market crashes) an investor could even lose money – even though he correctly guessed the direction of the market over the investment period.

Let's look at an example from the financial crisis of 2008. If you purchased an inverse ETF, such as the ProShares UltraShort -2X ETF (DXD) which is based on the Dow Jones Industrial Average (DJIA) at the market close on October 23, 2008, and sold it on March 04, 2009, the DJIA would have fallen 21% as shown in Table 1.

Table 1: Even though the DJIA lost 21%, the inverse, -2X ETF -- rather than generating a handsome profit – also incurred a significant loss.

DATE	DJIA	%change	DXD	%change
23-Oct-08	8691.25		94.53	
04-Mar-09	6875.84	-21%	79.00	-16%

At first blush one would expect that the investment in the DXD should yield a 42% profit for the nearly five-month period. But, in fact, the investment would have yielded a loss of 6% as shown in the table.

This was not a fault of ProShares. The same would have occurred with the other leveraged index funds on the market. The cause of this shocking result was the path dependence that results from the structure of all current leveraged ETFs. This path dependence limits investing in such instruments only to speculators. Because most current

leveraged index ETFs in the US market re-index on a daily basis, they are best used for daily or intraday trading. They are not structured for a longer-term, buy-and-hold strategy. This also tends to make them unsuitable for retirement accounts. In fact, many brokers explicitly preclude their clients from employing such investments in their retirement accounts.

But wouldn't it be great to be able to purchase a 2x ETF as part of a 401(k) account and watch it grow over one's working career? Certainly, there might be some squeamishness during market downturns, but the major indices are designed to rise over time. As such, they have always righted themselves and gone on to new highs. If only a fund could be structured that was not path dependent!

4. A Solution

Recognizing this problem, we were frustrated by the inability of the market to offer a leveraged index ETF that we could purchase as part of our 401(k) program. Accordingly, we set out to develop a fund structure that provided the benefits of leverage, that can be used in retirement accounts (i.e., without the risk of margin calls), that was not path dependent. And we succeeded! We call the resulting family of fund models ΦFunds™.

If we made the same trade illustrated in Table 1 using an inverse 2X ΦFund™ we would obtain the results illustrated in Table 2.

Table 2: In contrast to the trade shown in Table 1 using a traditional -2X leveraged index fund, rather than losing 6%, the -2X ΦFund™ obtains a nominal return of 55% -- exactly what would be expected from its leverage.

DATE	S&P500	Nominal change	%change	-2X ΦFund	Nominal change	%change
17-Oct-08	985.40			985.40		
04-Mar-09	712.87	-272.53	-28%	1530.46	545.06	55%

As can be seen from the table, the return of the ΦFund™ structure is 55% -- in line with expectations of a -2X leveraged fund given the change in the underlying index of -28%. This stands in marked contrast to the performance of traditional -2 leveraged funds which lost money during this period. (Actual returns would be slightly less because of fund manager's fees, but even with a high fee of 2% per year, the profit would be quite healthy.

Not being fund managers, ourselves, there is not yet an implementation of this fund structure. But we have extensively modeled our fund structure and back-tested it under all market conditions for both the S&P500 and the DJI going back several decades. The fund performed as expected: if the market was up, the fund was up; if the market was down, the fund was down; and when the index was neutral, the fund was neutral. We performed tests under all market conditions through the Great Depression, the Internet Bubble, the 2008 mortgage crisis, and all markets in between.

That the fund performed as it did is actually a mathematical certainty. Once a ΦFund™ is created it's value becomes a function of the underlying index. This is to say, that for any value of the underlying index there is one, and only one, value for the ΦFund™. Because the value of the ΦFund™ is a function, it is possible to create a table that provides the exact value of the ΦFund™ for any value of the underlying index. Over time, the only

variation will result only from the reduction prompted by the expenses and management fee of the fund manager.

We show the results of both 2X and -2X ΦFunds™ based on the S&P500 in Table 3. Table 3 measures performance of an investment made on the first trading day of the volatile year of 2008: January 2. We then took measurements on the first trading day of each subsequent month.

Table 3: An example investment in both a 2X ΦFund™ and a -2X ΦFund™ based on the S&P500 for the volatile year of 2008 shows that both retain fidelity with the underlying S&P500 index throughout the year.

	S&P500 Index			2X ΦFund™			-2X ΦFund™		
DATE	S&P500	Nominal change	Cumulative % return	2X ΦFund™	Nominal change	Cumulative % return	-2X ΦFund™	Nominal change	Cumulative % return
Jan-08	1447.16			1447.16			1447.16		
Feb-08	1395.42	-51.74	-3.6%	1343.68	-103.48	-7.2%	1550.64	103.48	7.2%
Mar-08	1331.34	-64.08	-8.0%	1215.52	-128.16	-16.0%	1678.80	128.16	16.0%
Apr-08	1370.18	38.84	-5.3%	1293.20	77.68	-10.6%	1601.12	-77.68	10.6%
May-08	1409.34	39.16	-2.6%	1371.52	78.32	-5.2%	1522.80	-78.32	5.2%
Jun-08	1385.67	-23.67	-4.2%	1324.18	-47.34	-8.5%	1570.14	47.34	8.5%
Jul-08	1284.91	-100.76	-11.2%	1122.66	-201.52	-22.4%	1771.66	201.52	22.4%
Aug-08	1260.31	-24.60	-12.9%	1073.46	-49.20	-25.8%	1820.86	49.20	25.8%
Sep-08	1277.58	17.27	-11.7%	1108.00	34.54	-23.4%	1786.32	-34.54	23.4%
Oct-08	1161.06	-116.52	-19.8%	874.96	-233.04	-39.5%	2019.36	233.04	39.5%
Nov-08	966.30	-194.76	-33.2%	485.44	-389.52	-66.5%	2408.88	389.52	66.5%
Dec-08	816.21	-150.09	-43.6%	185.26	-300.18	-87.2%	2709.06	300.18	87.2%

To make comparisons easy, we gave the fund the same value as the closing price of the S&P500 on this day. These results may be a bit misleading, because only in this scenario where the fund is purchased on the day it is issued will the percentage returns be the same as the nominal returns upon which the fund model is based. Investing at any other time, the cumulative percentage returns will not be exactly 2X or -2X. But it will still be the case that:

1. If the index is up, the fund will be up.
2. If the index is down, the fund will be down.
3. If the index is neutral the fund will be neutral.

At no time will a favorable change in the underlying index result in an unfavorable outcome.

To illustrate this divergence in the percentage return, we show the same investment depicted in Table 4. Instead of buying the fund on January 2, we illustrate the impact of making the investment on March 3.

Table 4: Making an investment on any day after the fund is initiated will cause the amount of leverage to vary. The cumulative percentage return will not be an exact multiple of the leverage factor times the return of the index. But the direction of the return (e.g., profit or loss) continues to maintain fidelity with the underlying index.

	S&P500 Index			2X ΦFund™			-2X ΦFund™		
DATE	S&P500	Nominal change	Cumulative % return	2X ΦFund™	Nominal change	Cumulative % return	-2X ΦFund™	Nominal change	Cumulative % return
Jan-08	1447.16			1447.16			1447.16		
Feb-08	1395.42	-51.74		1343.68	-103.48		1550.64	103.48	
Mar-08	1331.34	-64.08		1215.52	-128.16		1678.80	128.16	
Apr-08	1370.18	38.84	2.9%	1293.20	77.68	6.4%	1601.12	-77.68	-4.6%
May-08	1409.34	39.16	5.9%	1371.52	78.32	12.8%	1522.80	-78.32	-9.3%
Jun-08	1385.67	-23.67	4.1%	1324.18	-47.34	8.9%	1570.14	47.34	-6.5%
Jul-08	1284.91	-100.76	-3.5%	1122.66	-201.52	-7.6%	1771.66	201.52	5.5%
Aug-08	1260.31	-24.60	-5.3%	1073.46	-49.2	-11.7%	1820.86	49.20	8.5%
Sep-08	1277.58	17.27	-4.0%	1108.00	34.54	-8.8%	1786.32	-34.54	6.4%
Oct-08	1161.06	-116.52	-12.8%	874.96	-233.04	-28.0%	2019.36	233.04	20.3%
Nov-08	966.30	-194.76	-27.4%	485.44	-389.52	-60.1%	2408.88	389.52	43.5%
Dec-08	816.21	-150.09	-38.7%	185.26	-300.18	-84.8%	2709.06	300.18	61.4%

Our structure obviously differs from the current structure. To avoid path dependence, we apply leverage to the **nominal** change in an index, not its **percentage** change. In this way, when the index is at a particular value X, the ETF would have a value Y. And even if the market goes through a major crisis, when the index returns to X, the ETF will have the exact value Y. There is a 1:1 correspondence of the ETF value to the underlying index for every value of the index. But the leverage will diminish if the fund is purchased at a price above its initial value and it will increase as the fund is purchased at a lesser value.

Essentially, the leveraged obtained becomes fixed at the time of purchased. The leverage factor is easily calculatable as

$$\textbf{Initial ΦFund™ leverage} * \frac{\textbf{Initial Offering price}}{\textit{Purchase price}}$$

This variance in the leverage is merely the result of a change in the denominator when calculating returns. The other two factors are fixed forever in time. A 2X initial leverage when the ΦFund™ was purchased at its initial offering price (e.g., $100), results in 2X leverage for the life of that investment. If later another investment is made when the ΦFund™ has risen to $120, the resulting leverage will be 1.66 (2*100/120) for the life of that investment. And another investment made when the ΦFund™ is valued at $80 will yield a leverage of 2.5 for the life of that investment.

And unlike current funds, when the index retraces itself, the ΦFund™ leverage will also retrace itself, enabling the fund to exactly retrace its value – even though different investments are obtaining different amounts of leverage. In this way, for any given value of the underlying index, the value of the fund will always be the same. Looking at Table 4 and the values on September 2, 2008, the S&P500 closed at 1277.58 and the model 2X ΦFund™ closed at 874.96. When the S&P eventually recovered to the 1277.58 level intraday in January 2011, the ΦFund™ would also recover to exactly 874.96.

As can be seen in Table 4, the cumulative return percentages of the 2X ΦFund™ are no longer exact multiples of the cumulative return of the underlying index, but remain completely predictable. They continue to reflect leveraged returns in the expected direction. The difference occurs merely because the 2X ΦFund™ was purchased when the index was at a price below the initial offering price. As such the nominal change in the 2X fund will be a greater percentage of this lower purchase price ($12258.90) than had it been purchased at the initial opening price ($13043.96). The leverage is altered by the ratio or the new index price to the new leveraged fund price. The formula for calculating leverage is

Initial leverage * (Current Index Value/Current Fund Price)

When the fund is purchased at its initial price, a 2X fund will retain the leverage of 2.0 [2.0*(13043.96/13043.96)]. For a purchase made on March 3, the actual leverage of the 2X ΦFund™ would be 2.14 [2.0*(12258.90/11473.84)]. Once purchased, however, the leverage does not change for the life of the investment. This can be seen in Table 5. The actual leverage shown in Table 5 can be quickly calculated by dividing the cumulative return of the ΦFund™ by the Cumulative return of the index for each period.

Table 5: Once an investment is made in a leveraged-index ΦFund™, the leverage remains constant through the life of the investment.

	DJIA		2X ΦFund		
DATE	DJIA	Cumulative % return	2X ΦFund	Cumulative % return	Actual Leverage
Jan-08	13043.96		13043.96		
Feb-08	12743.19		12442.42		
Mar-08	12258.90		11473.84		
Apr-08	12654.36	**3.2%**	12264.76	**6.9%**	**2.14**
May-08	13010.00	**6.1%**	12976.04	**13.1%**	**2.14**
Jun-08	12503.82	**2.0%**	11963.68	**4.3%**	**2.14**
Jul-08	11382.26	**-7.2%**	9720.56	**-15.3%**	**2.14**
Aug-08	11326.32	**-7.6%**	9608.68	**-16.3%**	**2.14**
Sep-08	11516.92	**-6.1%**	9989.88	**-12.9%**	**2.14**
Oct-08	10831.07	**-11.6%**	8618.18	**-24.9%**	**2.14**
Nov-08	9319.83	**-24.0%**	5595.70	**-51.2%**	**2.14**
Dec-08	8149.09	**-33.5%**	3254.22	**-71.6%**	**2.14**

For an inverse ΦFund™, the same logic applies. Purchased at a higher price ($14614.08) than the initial offering price ($13042.96), the leverage on the -2X ΦFund™ declines to -1.68 [-2.0*(12258.90/14614.08)], and remains constant throughout the life of that investment.

Because, in general, indices such as the DJIA are designed to rise over time, it can be expected that a ΦFund™ with a positive initial leverage (e.g., 2X or 3X) will experience significant declines in leverage available for new investments over time. The leverage will always be present, but at levels just above 1.0, the appeal of the fund to investors would likely diminish. A way for a fund manager to address this would be to issue a new fund beginning at the higher current value. New investors could then invest in the new fund to obtainn higher levels of leverage.

To view a longer-term investment, we capture an investment in an DJIA-based ΦFund™ with 2X initial leverage beginning in January, 2008, and display the value of the leveraged ΦFund™ for the first trading day of every succeeding year in Table 6.

Table 6: A long-term investment in a 2X leveraged-index ΦFund™ based on the Dow Jones Industrial Average continues to generate outsized (2X) returns over the long term, allowing conservative investors to benefit from such returns (absent fund manager expenses and fees).

	Dow Jones Industrial Avg			2X ΦFund		
DATE	DJIA	Nominal change	Cumulative % return	2X ΦFund	Nominal change	Cumulative % return
2008-01-02	13043.96			13043.96		
2009-01-02	9034.69	-4009.27	-30.7%	5025.42	-8018.54	-61.5%
2010-01-04	10583.96	1549.27	-18.9%	8123.96	3098.54	-37.7%
2011-01-03	11670.75	1086.79	-10.5%	10297.54	2173.58	-21.1%
2012-01-03	12397.38	726.63	-5.0%	11750.80	1453.26	-9.9%
2013-01-02	13412.55	1015.17	2.8%	13781.14	2030.34	5.7%
2014-01-02	16441.35	3028.80	26.0%	19838.74	6057.60	52.1%
2015-01-02	17832.99	1391.64	36.7%	22622.02	2783.28	73.4%
2016-01-04	17148.94	-684.05	31.5%	21253.92	-1368.10	62.9%
2017-01-03	19881.76	2732.82	52.4%	26719.56	5465.64	104.8%
2018-01-02	24824.01	4942.25	90.3%	36604.06	9884.50	180.6%
2019-01-02	23346.24	-1477.77	79.0%	33648.52	-2955.54	158.0%
2020-01-02	28868.80	5522.56	121.3%	44693.64	11045.12	242.6%
2021-01-04	30223.89	1355.09	131.7%	47403.82	2710.18	263.4%
2022-01-03	36585.06	6361.17	180.5%	60126.16	12722.34	361.0%
2023-01-03	33136.37	-3448.69	154.0%	53228.78	-6897.38	308.1%

As can be seen from Table 6, the ΦFund™ purchased at the initiation of a ΦFund™ on January 2, 2008 would double both the nominal and percentage return on the purchase of a basic index fund. This is illustrated graphically in Figure 1.

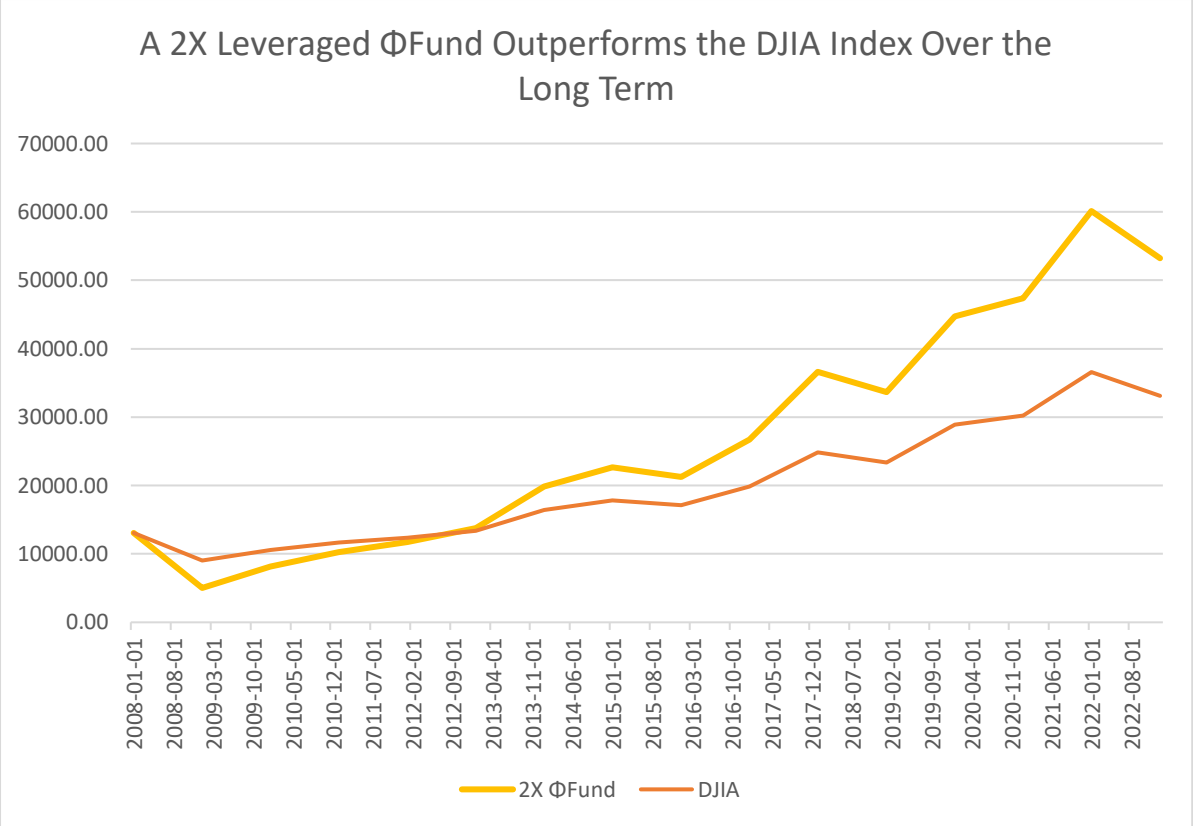

Figure 1: Because the 2X leveraged Fund maintains fidelity with its underlying index, it can be expected to provide outsized returns for underlying indices that always rise over time such as the Dow Jones Industrial Average (shown here) and the S&P500.

To demonstrate that such returns are not unique to the DJIA, we show results for a model 2X leverage-index ΦFund™ based on the S&P500 in Table 7.

Table 7: A long-term investment in 2X leverage ΦFund™, based on the S&P500 would yield a return double that of the underlying DJIA Index (before the reduction of fund management fees).

	Dow Jones Industrial Avg			2X ΦFund		
DATE	DJIA	Nominal change	Cumulative % return	2X ΦFund	Nominal change	Cumulative % return
2008-01-02	13043.96			13043.96		
2009-01-02	9034.69	-4009.27	-30.7%	5025.42	-8018.54	-61.5%
2010-01-04	10583.96	1549.27	-18.9%	8123.96	3098.54	-37.7%
2011-01-03	11670.75	1086.79	-10.5%	10297.54	2173.58	-21.1%
2012-01-03	12397.38	726.63	-5.0%	11750.80	1453.26	-9.9%
2013-01-02	13412.55	1015.17	2.8%	13781.14	2030.34	5.7%
2014-01-02	16441.35	3028.80	26.0%	19838.74	6057.60	52.1%
2015-01-02	17832.99	1391.64	36.7%	22622.02	2783.28	73.4%
2016-01-04	17148.94	-684.05	31.5%	21253.92	-1368.10	62.9%
2017-01-03	19881.76	2732.82	52.4%	26719.56	5465.64	104.8%
2018-01-02	24824.01	4942.25	90.3%	36604.06	9884.50	180.6%
2019-01-02	23346.24	-1477.77	79.0%	33648.52	-2955.54	158.0%
2020-01-02	28868.80	5522.56	121.3%	44693.64	11045.12	242.6%
2021-01-04	30223.89	1355.09	131.7%	47403.82	2710.18	263.4%
2022-01-03	36585.06	6361.17	180.5%	60126.16	12722.34	361.0%
2023-01-03	33136.37	-3448.69	154.0%	53228.78	-6897.38	308.1%

Purchases made at any time other than fund initiation would still produce outsized returns as shown in Table 8**Error! Reference source not found.**.

Table 8: Because the returns of the leveraged ΦFund™ are not path dependent, they maintain fidelity with the underlying index eternally. No re-indexing is ever required.

DATE	DJIA	Nominal change	Cumulative % return	2X ΦFund	Nominal change	Cumulative % return
2008-01-02	13043.96			13043.96		
2008-03-03	12264.36	-779.60		11484.76	-1559.20	
2009-01-02	9034.69	-3229.67	-26.3%	5025.42	-6459.34	-56.2%
2010-01-04	10583.96	1549.27	-13.7%	8123.96	3098.54	-29.3%
2011-01-03	11670.75	1086.79	-4.8%	10297.54	2173.58	-10.3%
2012-01-03	12397.38	726.63	1.1%	11750.80	1453.26	2.3%
2013-01-02	13412.55	1015.17	9.4%	13781.14	2030.34	20.0%
2014-01-02	16441.35	3028.80	34.1%	19838.74	6057.60	72.7%
2015-01-02	17832.99	1391.64	45.4%	22622.02	2783.28	97.0%
2016-01-04	17148.94	-684.05	39.8%	21253.92	-1368.10	85.1%
2017-01-03	19881.76	2732.82	62.1%	26719.56	5465.64	132.7%
2018-01-02	24824.01	4942.25	102.4%	36604.06	9884.50	218.7%
2019-01-02	23346.24	-1477.77	90.4%	33648.52	-2955.54	193.0%
2020-01-02	28868.80	5522.56	135.4%	44693.64	11045.12	289.2%
2021-01-04	30223.89	1355.09	146.4%	47403.82	2710.18	312.8%
2022-01-03	36585.06	6361.17	198.3%	60126.16	12722.34	423.5%
2023-01-03	33136.37	-3448.69	170.2%	53228.78	-6897.38	363.5%

Error! Reference source not found. assumes the same purchase date of March 3, 2008 as shown in Table 4. Because the purchase price ($11484.76) was lower than the initial purchase price ($13043.96), the returns of the fund reflect a 2.2X leverage, returning more than 350% over the nearly 15-year period, versus the 170% return of the DJIA over the same period (exclusive of management fees and expenses).

To further emphasize the path independence of ΦFunds™ we modeled the returns of an investment made on the first trading day of each year held until the first trading day of 2023 and compared the performance of the 2X ΦFund™ to that of the DJIA in Table 9.

Table 9: Cumulative returns for investments made each year shows that, regardless of when the investment was made, the ΦFund™ (2X in this example) maintains its fidelity with the underlying index and earns leveraged returns over time.

	DJIA			2X ΦFund			
Purchase Date	DJIA	Cumulative Nominal Change	Cumulative % return to 2023-01-03	2X ΦFund	Cumulative Nominal Change	Cumulative % return to 2023-01-03	Leverage Obtained
2008-01-02	13043.96	20092.41	154.0%	13043.96	40184.82	308.1%	2.0
2009-01-02	9034.69	24101.68	266.8%	5025.42	48203.36	959.2%	3.6
2010-01-04	10583.96	22552.41	213.1%	8123.96	45104.82	555.2%	2.6
2011-01-03	11670.75	21465.62	183.9%	10297.54	42931.24	416.9%	2.3
2012-01-03	12397.38	20738.99	167.3%	11750.80	41477.98	353.0%	2.1
2013-01-02	13412.55	19723.82	147.1%	13781.14	39447.64	286.2%	1.9
2014-01-02	16441.35	16695.02	101.5%	19838.74	33390.04	168.3%	1.7
2015-01-02	17832.99	15303.38	85.8%	22622.02	30606.76	135.3%	1.6
2016-01-04	17148.94	15987.43	93.2%	21253.92	31974.86	150.4%	1.6
2017-01-03	19881.76	13254.61	66.7%	26719.56	26509.22	99.2%	1.5
2018-01-02	24824.01	8312.36	33.5%	36604.06	16624.72	45.4%	1.4
2019-01-02	23346.24	9790.13	41.9%	33648.52	19580.26	58.2%	1.4
2020-01-02	28868.80	4267.57	14.8%	44693.64	8535.14	19.1%	1.3
2021-01-04	30223.89	2912.48	9.6%	47403.82	5824.96	12.3%	1.3
2022-01-03	36585.06	-3448.69	-9.4%	60126.16	-6897.38	-11.5%	1.2
2023-01-03	33136.37			53228.78			

Table 9 also illustrates the gradual reduction in leverage obtained for investments as the underlying index continues to increase. The exactly 2X leverage obtained at the instantiation of the imaginary ΦFund™ in January 2008 rose to 4.5 as the market collapsed later in that year, but then began a steady decline as the value of the fund continued to rise over time. After ten years, it was less than 1.5X. At such a point, a real fund might decide to issue a new fund, creating a new leverage baseline of 2X in 2018. But even with the decline in leverage as the value of the DJIA continued to grow, an investment in a 2X ΦFund™ would have outperformed the S&P in every year except 2022 when the index lost ground and has not yet had time to recover. That's 14 of the past 15 years.

4.1. Solving a minor glitch

Using the nominal value, rather than a percentage value could have the potential to cause the fund value to go negative. If an index begins at 100 and a -2X ΦFund™ also begins at 100 and the index rises 51 points to 151, the -2X ΦFund™ should decline 102 points which would give it an unacceptable valuation of -2. We resolve this by creating a tapering of the leverage below a fixed threshold. If the threshold is reached, the leverage declines. Using zero as an asymptote, the leverage changes continuously so that a zero valuation is never attained. Equally important, if/when the fund value begins to rise the leverage follows the identical path on its way up. In this way, there remains a 1:1 correlation of the value of the fund to the value of the underlying index. In fact, the values of the fund can be published to show the value of the fund for any value of the underlying index. And because it is not path dependent, the values will remain constant over all periods of time.

Our work has received US patent US 8,296,214. It has also been granted in several other major world markets.

5. Conclusion

It is possible to create a leveraged index fund (mutual fund or ETF) that maintains fidelity with its underlying index under all market conditions for all time frames. The resulting fund is not path dependent and is suitable for long-term portfolios including retirement accounts. If the fund is based on an index that is designed to rise over time (e.g., the S&P500 and the Dow Jones Industrial Average), a leveraged ΦFund™ can be expected to consistently provide outsized returns over the period of a long-term investment using a buy-and-hold strategy. It, therefore, would be suitable for a retirement account, such as a 401(k), if the account owner still anticipated many more years of life.

6. Bibliography

Elisabeth Kashner, "The Truth About Leveraged ETF Returns", https://www.etf.com/sections/blog/21176-the-truth-about-leveraged-etf-returns.html, February 11, 2014.

Marco Avellaneda and Stanley Zhang, "Path-Dependence of Leveraged ETF Returns" SIAM Journal on Financial MathematicsVol. 1, Iss. 1 (2010)10.1137/090760805, https://math.nyu.edu/~avellane/SIAMLETFS.pdf.pdf, 2010.

Jian Zhang, “Path-dependence properties of leveraged exchange-traded funds: Compounding, volatility and option pricing”, https://www.semanticscholar.org/paper/Path-dependence-properties-of-leveraged-funds%3A-and-Zhang/08a74f6420b644985fd2737066f0ba51acb6ae95, 2010.

Jeff Stollman, “In Search of Leveraged Returns for Long-term Investment”, http://www.phi-funds.com/2015/05/19/in-search-of-leveraged-returns-for-long-term-investment/, May 19, 2015.